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Color dielectric model with two scalar fields

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Abstract. $SU(2)$ Yang–Mills theory coupled in a non-minimal way to two scalar fields is discussed. For the massless scalar fields a family of finite energy solutions generated by an external, static electric charge is found. Additionally, there is a single solution which can be interpreted as a confining one. Similar solutions have been obtained in the magnetic sector. In the case of massive scalar fields the Coulomb problem is investigated. We find that asymptotic behavior of the fields can also, for some values of the parameters of the model, give confinement of the electric charge. Quite interestingly one glueball–meson coupling gives the linear confining potential. Finally, it is shown how, for one non-dynamical scalar field, we can derive the color dielectric generalization of the Pagels–Tomboulis model.

1 Introduction

There exist several attempts to understand the dynamics of gluonic fields in the non-perturbative region. Among them color dielectric models seem to be especially promising [1]. Although none of them describes the full range of observable phenomena in a satisfactory way, they are simple enough to carry out an extensive study of classical solutions and provide guidelines for constructing the correct effective theory for non-perturbative QCD.

A class of color dielectric-like models was proposed by Dick in the late 90's. It was inspired by the low-energy limit of certain string theories. In the simplest case, with one scalar field coupled non-minimally to the $SU(2)$ gauge field, Dick models reproduce a broad family of quark–antiquark confining potentials. As it has been pointed out in our previous papers [2] one can easily fit potentials originating from these models to phenomenological ones, obtained from heavy quarkonia spectroscopy.

The correct potential between quarks is of course only one in the whole family of factors to be considered and reproduced by an effective gluodynamics theory. An acceptable model should also exclude colored states from the physical spectrum, reproduce the energy-momentum tensor trace anomaly as well as predict the existence of massive particles built entirely of gluonic fields – glueballs. The latter issue is particularly challenging and was addressed by several authors in the last few months (cf. e.g. [3,4]). A glueball-like interpretation has been also proposed for the dilaton field in the Dick model [5,6]. However, so far the status of the glueball state in the framework of color dielectric models is not clear.

The most natural generalization of the Dick model emerges when the dilaton field is replaced by a pair of two scalars. A theory of this kind was recently studied by Bazeia et al. [7] in the context of the confining potential in a system of two planar domain walls. They concluded that a theory with two scalars is generally better suited to a description of anti-screening effects. As we will show below, a color dielectric model with two scalars possesses also other striking features. Surprisingly the observed effects are not a simple superposition of well-known results for one scalar. In particular, a non-minimal two scalar–gauge coupling has a glueball as well as a meson interpretation and simultaneously gives a linear confining potential for external sources. For non-dynamical scalar fields it is also equivalent to the well-known Pagels–Tomboulis model and its generalized version, which correctly reproduces the trace anomaly in the language of effective models.

The plan of this paper is the following. In the next section we introduce and briefly justify the most general form of the model discussed. In Sect. 3 a standard analysis of the Coulomb problem for an external source in the case of a massless scalar field is carried out, which eventually leads to confining behavior. It also allows us to obtain bounds on the parameters and rule out the whole family of models which do not lead to the correct confining potential. Afterwards we deal with magnetic monopoles and discuss their possible application to glueball physics. In Sect. 4 we present a similar discussion for massive scalars. Finally, non-dynamical scalars are studied in Sect. 5.

2 The model

In the present paper we focus on the following density of the Lagrange function:

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$$L = -\frac{1}{4}\sigma(\phi,\psi)F^{c\mu\nu}F^{c}_{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\psi\partial^\mu\psi - \frac{1}{2}m_\phi^2\phi^2 - \frac{1}{2}m_\psi^2\psi^2 - V(\phi,\psi), \quad (1)$$

where the coupling between scalar and gauge fields is chosen to be

$$\sigma(\phi,\psi) = \left(\frac{\phi}{\Lambda}\right)^a \left(\frac{\psi}{\Lambda}\right)^b. \quad (2)$$

Here Λ is a dimensional constant: $[\Lambda] = \text{cm}^{-1}$ and a, b are positive parameters. A particular form of the potential $V(\phi,\psi)$, fixing the asymptotic value of the scalar fields, will be specified later.

We would like to notice that all analytical solutions obtained below can also be found for the more general coupling function

$$\sigma(\phi,\psi) = \frac{\left(\frac{\phi}{\Lambda}\right)^a \left(\frac{\psi}{\Lambda}\right)^b}{1+\left(\frac{\phi}{\Lambda}\right)^a \left(\frac{\psi}{\Lambda}\right)^b}. \quad (3)$$

Due to the fact that for $\phi \to \infty$, $\psi \to \infty$ the gauge part of the model takes the standard Yang–Mills form, i.e., $\sigma \to 1$, the short range behavior of the fields changes. For instance, the electric field falls as r^{-2} for $r \to 0$. One can check that in the presented solutions the additional, well-known Coulomb term will appear. However, such a form of the coupling does not modify the most interesting long range behavior of the fields. Thus we will neglect the denominator in the coupling function, that is, restrict our discussion to the dielectric function given by (2).

3 The massless scalar fields

Firstly we analyze the simplest case where the scalar fields are massless. Additionally, the potential term is neglected. Due to that, as we will show in the next subsection, the asymptotic values of the scalar fields can be arbitrarily large. These values correspond to the so-called dilaton and modulus charge [8]. The Lagrange density has the following form:

$$L = -\frac{1}{4}\left(\frac{\phi}{\Lambda}\right)^a \left(\frac{\psi}{\Lambda}\right)^b F^{c\mu\nu}F^{c}_{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\psi\partial^\mu\psi. \quad (4)$$

The pertinent equations of motion read

$$D_\mu\left(\left(\frac{\phi}{\Lambda}\right)^a \left(\frac{\psi}{\Lambda}\right)^b F^{c\mu\nu}\right) = j^{c\nu} \quad (5)$$

and

$$\partial_\mu\partial^\mu\phi = -\frac{a}{4\Lambda}\left(\frac{\phi}{\Lambda}\right)^{a-1} \left(\frac{\psi}{\Lambda}\right)^b F^{c\mu\nu}F^{c}_{\mu\nu}, \quad (6)$$

$$\partial_\mu\partial^\mu\psi = -\frac{b}{4\Lambda}\left(\frac{\phi}{\Lambda}\right)^{a} \left(\frac{\psi}{\Lambda}\right)^{b-1} F^{c\mu\nu}F^{c}_{\mu\nu}, \quad (7)$$

where $j^{c\nu}$ is an external current.

3.1 Electric solutions

Let us now investigate the Coulomb problem, i.e. we will find the field configuration generated by an external, static, point-like electric source:

$$j^{c\mu} = 4\pi q\delta(r)\delta^{c3}\delta^{\mu 0}. \quad (8)$$

Moreover, for simplicity, the source is set to be Abelian. We would like to stress that restriction to the Abelian case is not essential. One can easily analyze the more general source with three non-zero color components. However, the results will be modified only by a multiplicative color-dependent constant (cf. [5]). The dependence on the spatial coordinates is identical. Because of that we remain in the Abelian sector. In order to work only with Abelian degrees of freedom one can put

$$A^c_\mu = A_\mu\delta^{c3},$$

and the field equations can be rewritten as follows:

$$\left[r^2\left(\frac{\phi}{\Lambda}\right)^a \left(\frac{\psi}{\Lambda}\right)^b E\right]' = 4\pi q\delta(r), \quad (9)$$

and

$$\nabla_r^2\phi = -\frac{aE^2}{2\Lambda}\left(\frac{\phi}{\Lambda}\right)^{a-1} \left(\frac{\psi}{\Lambda}\right)^b, \quad (10)$$

$$\nabla_r^2\psi = -\frac{bE^2}{2\Lambda}\left(\frac{\phi}{\Lambda}\right)^{a} \left(\frac{\psi}{\Lambda}\right)^{b-1}. \quad (11)$$

Here $E^{ci} = -F^{c0i}$ and $\mathbf{E}^c = \mathbf{E}\delta^{c3}$ i.e. the electric field points in the same color direction as the source. We have also assumed spherical symmetry of the problem $\mathbf{E} = E(r)\hat{r}$. The prime stands for differentiation with respect to r. Using the Gauss law (9) one can express the electric field in terms of the scalar fields:

$$E = \frac{q}{r^2}\left(\frac{\phi}{\Lambda}\right)^{-a} \left(\frac{\psi}{\Lambda}\right)^{-b}. \quad (12)$$

One can treat this field as the standard Coulomb field in a very non-standard medium. Indeed, it can be written as $E = \frac{q_{\text{eff}}}{r^2}$ where q_{eff} is an effective charge strongly dependent on the scalar fields. Now, we insert it into the equations for the scalars and get

$$\ddot{\phi} = -\frac{aq^2}{2\Lambda}\left(\frac{\phi}{\Lambda}\right)^{-a-1} \left(\frac{\psi}{\Lambda}\right)^{-b} \quad (13)$$

and

$$\ddot{\psi} = -\frac{bq^2}{2\Lambda}\left(\frac{\phi}{\Lambda}\right)^{-a} \left(\frac{\psi}{\Lambda}\right)^{-b-1}, \quad (14)$$

where a new variable $x = 1/r$ has been introduced. Here a dot denotes differentiation with respect to x. We look for solutions to the equations (13) and (14) in a power-like form. Namely

$$\phi(x) = A\Lambda\left(\frac{x}{\Lambda} + \frac{x_0}{\Lambda}\right)^n \quad (15)$$

and

$$\psi(x) = B\Lambda \left(\frac{x}{\Lambda} + \frac{x_0}{\Lambda}\right)^m, \tag{16}$$

where the constants m, n, A, B and x_0 are yet to be determined. After some easy algebra one can find that

$$n = m = \frac{2}{2+a+b} \tag{17}$$

and

$$A = a^{\frac{1}{2+a+b}} \left(\frac{b}{a}\right)^{-\frac{b}{2(2+a+b)}} \left(\frac{q^2(2+a+b)^2}{4(a+b)}\right)^{\frac{1}{2+a+b}}, \tag{18}$$

$$B = b^{\frac{1}{2+a+b}} \left(\frac{a}{b}\right)^{-\frac{a}{2(2+a+b)}} \left(\frac{q^2(2+a+b)^2}{4(a+b)}\right)^{\frac{1}{2+a+b}}. \tag{19}$$

Finally, we have found the following family of solutions to the Coulomb problem, labeled by the positive parameter β_0:

$$\phi(r) = A\Lambda \left(\frac{1}{r\Lambda} + \frac{1}{\beta_0}\right)^{\frac{2}{2+a+b}}, \tag{20}$$

$$\psi(r) = B\Lambda \left(\frac{1}{r\Lambda} + \frac{1}{\beta_0}\right)^{\frac{2}{2+a+b}}. \tag{21}$$

The electric field takes the form

$$E = \frac{q}{r^2} A^{-a} B^{-b} \left(\frac{1}{r\Lambda} + \frac{1}{\beta_0}\right)^{\frac{-2a-2b}{2+a+b}}. \tag{22}$$

Here we have put $\beta_0 = \frac{\Lambda}{x_0}$. Then the corresponding energy density reads

$$\epsilon = \frac{1}{2}\left(q^2 A^{-a} B^{-b} + \frac{4(A^2+B^2)}{(2+a+b)^2}\right)\frac{1}{r^4} \times \left(\frac{1}{r\Lambda} + \frac{1}{\beta_0}\right)^{-\frac{2a+2b}{2+a+b}}. \tag{23}$$

Integrating (23) one finds that the total energy of the fields generated by the external static electric charge is finite if $a+b>2$, and it is given by the formula

$$\mathcal{E} = \Lambda \frac{a+b+2}{a+b-2}\frac{1}{2}\left(q^2 A^{-a} B^{-b} + \frac{4(A^2+B^2)}{(2+a+b)^2}\right)\beta_0^{\frac{a+b-2}{a+b+2}}. \tag{24}$$

Of course, there are also solutions with a negative parameter β_0. However, such field configurations are not regular for all r. There is a singularity at $r \to \beta$. Due to that such solutions are usually interpreted as electric black hole solutions with $r > \beta_0$ [8].

In the case of the family of solutions with finite energy we can define some additional numbers, the so-called dilaton (scalar) and gauge (electric) charges, which characterize the asymptotic behavior of the solutions. Using the standard definitions we obtain

$$Q = r^2 E(r)|_{r\to\infty} = q A^{-a} B^{-b} \beta_0^{\frac{2a+2b}{2+a+b}}, \tag{25}$$

for the electric charge and

$$D_\phi = -r^2 \left.\frac{\mathrm{d}\phi}{\mathrm{d}r}\right|_{r\to\infty} = \frac{2}{2+a+b} A \beta_0^{\frac{a+b}{2+a+b}}, \tag{26}$$

$$D_\psi = -r^2 \left.\frac{\mathrm{d}\psi}{\mathrm{d}r}\right|_{r\to\infty} = \frac{2}{2+a+b} B \beta_0^{\frac{a+b}{2+a+b}} \tag{27}$$

for the scalar charges.

One can notice that the scalar charges and gauge charge are not independent. They are connected by the simple relation

$$\frac{D_\phi D_\psi}{Q} = q\frac{a+b}{\sqrt{ab}}. \tag{28}$$

The finite energy solutions to the Coulomb problem can be interpreted as screening configurations. The fields generated by a fixed electric charge can have an arbitrarily small energy. This phenomenon is known from the standard non-Abelian Yang–Mills theory where the non-Abelian contents of the gauge field lowers the total energy [9]. Here it occurs even in the Abelian part of the model. This suggests that scalar fields can probably represent the non-Abelian part of the gluonic sector of QCD [10].

In addition to the family of the finite energy field configurations, there is an unique infinite energy solution

$$\phi(r) = A\Lambda \left(\frac{1}{r\Lambda}\right)^{\frac{2}{2+a+b}}, \tag{29}$$

$$\psi(r) = B\Lambda \left(\frac{1}{r\Lambda}\right)^{\frac{2}{2+a+b}} \tag{30}$$

and

$$E = q A^{-a} B^{-b} \Lambda^2 \left(\frac{1}{r\Lambda}\right)^{\frac{4}{2+a+b}}. \tag{31}$$

Then the electric potential reads

$$U = q\frac{a+b+2}{a+b-2} A^{-a} B^{-b} \Lambda \left(\frac{1}{r\Lambda}\right)^{\frac{2-a-b}{2+a+b}} \tag{32}$$

for $a+b \neq 2$ and

$$U = q\Lambda A^{-a} B^{-b} \ln r\Lambda \tag{33}$$

for $a+b=2$. The pertinent energy density is given as

$$\epsilon = \frac{1}{2}\left(q^2 A^{-a} B^{-b} + \frac{4(A^2+B^2)}{(2+a+b)^2}\right)\frac{1}{r^4}\left(\frac{1}{r\Lambda}\right)^{\frac{-2a-2b}{2+a+b}}. \tag{34}$$

Obviously, the corresponding total energy is infinite. However, for $a+b \geq 2$ the infiniteness of the total energy has its origin in the long range behavior of the fields. It is unlikely to be the standard Coulomb potential in the classical electrodynamics where the energy is infinite because of the singularity of the potential at $r=0$. This effect has been used in many QCD motivated theories to model confinement of quarks on the classical level. The confining electric potential obtained here does not diverge stronger than linearly with distance r. That is in agreement with the famous Seiler constraints for the quark–anti-quark potential [11]. One can observe that the standard linear potential emerges only in the limit $a+b \to \infty$ which cannot be implemented on the Lagrangian level. This means that the model with two scalars does not describe the linear potential. However, it can be approximated with arbitrary accuracy by taking a sufficiently large value of the parameters a and b. In spite of the fact that our model does not possess the linear potential, it can still be physically interesting. In fact, it gives confining potentials which can be compared with many phenomenological potentials obtained from fits to the charmonium and bottomium states. For example, for $a+b=6$ one obtains the confining part of the Zalewski–Motyka potential [12]

$$U_{\mathrm{ZM}} = C_1 \left(\sqrt{r} - \frac{C_2}{r} \right), \tag{35}$$

where $C_1 \simeq 0.71\,\mathrm{GeV}^{\frac{1}{2}}$ and $C_2 \simeq 0.46\,\mathrm{GeV}^{\frac{3}{2}}$. Analogously, the Martin potential [13] is reproduced for $a+b=\frac{22}{9}$.

As long as the asymptotic values of the scalar fields are not fixed, i.e. the model does not contain a potential term for scalars; the confining and finite energy solutions appear simultaneously. In order to get rid of the screening field configurations and to preserve the confining solution one has to add a particular potential therm. Let us, for instance, choose it in the following form, which enables us to obtain analytical solutions:

$$V(\phi,\psi) = \Lambda^4 \sum_{i=1}^{N} \alpha_i \left(\frac{\phi}{\Lambda}\right)^{4+a+b-\beta_i} \left(\frac{\psi}{\Lambda}\right)^{\beta_i}, \tag{36}$$

where every parameter β_i fulfills $0<\beta_i<4+a+b$. The constants α_i are restricted by the condition that the potential must be positively valued and has a global minimum at $\phi=\psi=0$. Then the equations for the scalar fields (13) and (14) can be rewritten in the form

$$\phi'' = -\frac{aq^2}{2\Lambda}\left(\frac{\phi}{\Lambda}\right)^{-a-1}\left(\frac{\psi}{\Lambda}\right)^{-b} + \frac{\Lambda^3}{x^4}\sum_i \alpha_i(4+a+b-\beta_i)\left(\frac{\phi}{\Lambda}\right)^{3+a+b-\beta_i}\left(\frac{\psi}{\Lambda}\right)^{\beta_i} \tag{37}$$

and

$$\psi'' = -\frac{bq^2}{2\Lambda}\left(\frac{\phi}{\Lambda}\right)^{-a}\left(\frac{\psi}{\Lambda}\right)^{-b-1} + \frac{\Lambda^3}{x^4}\sum_i \alpha_i\beta_i\left(\frac{\phi}{\Lambda}\right)^{4+a+b-\beta_i}\left(\frac{\psi}{\Lambda}\right)^{\beta_i-1}. \tag{38}$$

The Gauss law remains unchanged and electric field can be expressed by the scalars via the formula (12). In the simplest case, when $N=1$, the solutions read

$$\phi(r) = \mathcal{A}\Lambda\left(\frac{1}{r\Lambda}\right)^{\frac{2}{2+a+b}}, \tag{39}$$

$$\psi(r) = \mathcal{B}\Lambda\left(\frac{1}{r\Lambda}\right)^{\frac{2}{2+a+b}} \tag{40}$$

and

$$E = q\Lambda^2\mathcal{A}^{-a}\mathcal{B}^{-b}\left(\frac{1}{r\Lambda}\right)^{\frac{4}{2+a+b}}, \tag{41}$$

where the constants $\mathcal{A},\mathcal{B}$ are given by the following set of algebraic equations:

$$\frac{2(a+b)}{(2+a+b)^2}\mathcal{A} = \frac{aq^2}{2}\mathcal{A}^{-a-1}\mathcal{B}^{-b} + \alpha(4+a+b-\beta)\mathcal{A}^{3+a+b-\beta}\mathcal{B}^{\beta}, \tag{42}$$

and

$$\frac{2(a+b)}{(2+a+b)^2}\mathcal{B} = \frac{bq^2}{2}\mathcal{A}^{-a}\mathcal{B}^{-b-1} + \alpha\beta\mathcal{A}^{4+a+b-\beta}\mathcal{B}^{\beta-1}. \tag{43}$$

One can notice that taking into account the potential (36) does not influence the functional dependence of the confining solutions. A modification of the Lagrangian is visible only in the constants $\mathcal{A}$ and $\mathcal{B}$. Because of the fact that the potential (36) has a unique minimum for the vanishing scalar fields the finite energy solutions (20)–(22) (asymptotically non-zero) can no longer have a finite energy. Moreover, one can check that such solutions are in contradiction with the equations of motion. In other words, the finite energy configurations disappear from our model.

3.2 Magnetic monopoles

In this subsection we find field solutions generated by the static magnetic monopole. In order to do this we adopt the well-known $SU(2)$ monopole ansatz

$$A_i^c = \epsilon_{cik}\frac{x^k}{r^2}(g-1),\ A_0^c = 0, \tag{44}$$

where $g=g(r)$ is an unknown function. Then the non-Abelian field equations read

$$\left[\left(\frac{\phi}{\Lambda}\right)^a\left(\frac{\psi}{\Lambda}\right)^b g'\right]' + \frac{1}{r^2}\left(\frac{\phi}{\Lambda}\right)^a\left(\frac{\psi}{\Lambda}\right)^b g\left(1-g^2\right) = 0, \tag{45}$$

and

$$\nabla_r^2\phi = \frac{a}{2\Lambda}\left[\frac{2g'^2}{r^2} + \frac{(g^2-1)^2}{r^4}\right]\left(\frac{\phi}{\Lambda}\right)^{a-1}\left(\frac{\psi}{\Lambda}\right)^b, \tag{46}$$

$$\nabla_r^2\phi = \frac{b}{2\Lambda}\left[\frac{2g'^2}{r^2} + \frac{(g^2-1)^2}{r^4}\right]\left(\frac{\phi}{\Lambda}\right)^{a}\left(\frac{\psi}{\Lambda}\right)^{b-1}. \tag{47}$$

Equation (45) possesses the trivial solution

$$g = 0. \tag{48}$$

In other words, we have found the Wu–Yang $SU(2)$ monopole. In the classical Yang–Mills theory this monopole has an infinite energy due to the singular behavior of the gauge field in the vicinity of the monopole. As we will see the scalar fields are able to "regularize" monopole solution.

After substituting (48) to the remaining field equations we can obtain the following family of solutions:

$$\phi = C\Lambda \left(\frac{1}{r\Lambda} + \frac{1}{\beta_0} \right)^{\frac{2}{2-a-b}} \tag{49}$$

and

$$\psi = D\Lambda \left(\frac{1}{r\Lambda} + \frac{1}{\beta_0} \right)^{\frac{2}{2-a-b}}, \tag{50}$$

where the constants read

$$C = a^{\frac{1}{2-a-b}} \left(\frac{b}{a} \right)^{\frac{b}{2(2-a-b)}} \left(\frac{q^2(2-a-b)^2}{4(a+b)} \right)^{\frac{1}{2-a-b}} \tag{51}$$

and

$$D = b^{\frac{1}{2-a-b}} \left(\frac{a}{b} \right)^{\frac{a}{2(2-a-b)}} \left(\frac{q^2(2-a-b)^2}{4(a+b)} \right)^{\frac{1}{2-a-b}}. \tag{52}$$

The energy density originating from (48)–(50) is

$$\epsilon = \frac{1}{2} \left(C^a D^b + \frac{4(C^2+D^2)}{(2-a-b)^2} \right) \frac{1}{r^4} \left(\frac{1}{r\Lambda} + \frac{1}{\beta_0} \right)^{\frac{2a+2b}{2-a-b}}. \tag{53}$$

As one could expect, for $a+b>2$ the total energy for the field generated by the Wu–Yang monopole is finite and reads

$$\mathcal{E} = \frac{a+b-2}{a+b+2} \frac{1}{2} \left(C^a D^b + \frac{4(C^2+D^2)}{(2-a-b)^2} \right) \beta_0^{\frac{a+b+2}{a+b-2}}. \tag{54}$$

Similarly to the case of the electric solution discussed above, the scalar fields surrounding the magnetic monopole possess dilaton charges, which take the values

$$D_\phi = \frac{2}{2-a-b} C \beta_0^{\frac{a+b}{a+b-2}} \tag{55}$$

and

$$D_\psi = \frac{2}{2-a-b} D \beta_0^{\frac{a+b}{a+b-2}}. \tag{56}$$

For a negative value of the parameter β_0 in the magnetic solutions (49) and (50) we find the magnetic counterpart of the electric black hole configuration presented previously.

It is widely accepted in the literature that finite energy magnetic solutions correspond to glueballs (see e.g. [14]) – effective particles in the low energy sector of gluodynamics. As we have shown the model (1) provides the whole family of glueball-like states.

In addition there is a solution with an infinite energy:

$$\phi(r) = C\Lambda \left(\frac{1}{r\Lambda} \right)^{\frac{2}{2-a-b}}, \tag{57}$$

$$\psi(r) = D\Lambda \left(\frac{1}{r\Lambda} \right)^{\frac{2}{2-a-b}} \tag{58}$$

for $a+b \neq 2$. One can check that, for $a+b>2$, the energy density diverges as $\sim r^\delta$ at the spatial infinity:

$$\epsilon = \frac{1}{2} \left(C^a D^b + \frac{4(C^2+D^2)}{(2-a-b)^2} \right) \frac{1}{r^4} \left(\frac{1}{r\Lambda} \right)^{\frac{2a+2b}{2-a-b}}. \tag{59}$$

Here δ takes values from 0 ($a+b \to \infty$) to infinity (when $a+b \to 2$).

Interestingly enough the electric as well as the magnetic solutions to the model (4) reveal an universal property. The solutions describing scalars are (up to a multiplicative constant) identical, even though these fields differently couple to the gauge field. i.e. with $a \neq b$. The energy density of the fields generated by the electric (magnetic) source behaves as if there has been only one (effective) scalar field σ coupled to $F^{c\mu\nu}F^c_{\mu\nu}$ invariant by σ^{a+b}.

To summarize, there are three sectors of solutions for the model defined by (4): screening, confining and glueball-like. They appear simultaneously i.e. for fixed parameters of the model one can derive a finite as well as an infinite energy solution. Because of the fact that the main aim of such models is to provide an effective description of the low energy features of QCD it is important to get rid of these non-physical screening solutions. It can easily be done by adding of the potential term (36). Unfortunately, this removes not only the finite energy electric solutions but also all the glueball-like solutions. This is highly unsatisfactory. The good candidate for an effective model describing the low energy QCD should model confinement and possess at least one glueball solution.

4 The massive scalar fields

In this section we turn to massive scalar fields. In effective models for the low energy gluodynamics such massive scalars are usually interpreted as scalar (effective) glueball and/or meson fields. This is unlikely in the massless case where scalar fields do not have any clear particle interpretation. Then the Lagrangian (1) takes the following form:

$$L = -\frac{1}{4} \left(\frac{\phi}{\Lambda} \right)^a \left(\frac{\psi}{\Lambda} \right)^b F^{c\mu\nu} F^c_{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \psi \partial^\mu \psi - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{2} m_\psi^2 \psi^2, \tag{60}$$

where m_ϕ, m_ψ are the masses of the scalars. Let us now consider the Coulomb problem for the massive model (60) and compare it with massless solutions. The pertinent equations of motion can be written as follows:

$$\left[r^2 \left(\frac{\phi}{\Lambda} \right)^a \left(\frac{\psi}{\Lambda} \right)^b E \right]' = 4\pi q \delta(r) \tag{61}$$

and

$$\nabla_r^2 \phi = -\frac{aE^2}{2\Lambda} \left(\frac{\phi}{\Lambda} \right)^{a-1} \left(\frac{\psi}{\Lambda} \right)^b + m_\phi^2 \phi, \tag{62}$$

$$\nabla_r^2\psi = -\frac{bE^2}{2\Lambda}\left(\frac{\phi}{\Lambda}\right)^a\left(\frac{\psi}{\Lambda}\right)^{b-1} + m_\psi^2\psi. \tag{63}$$

Obviously, the additional massive term for the scalars does not inflect the Gauss law. Thus, the electric field is given in terms of the scalar fields in the following way:

$$E = \frac{q}{r^2}\left(\frac{\phi}{\Lambda}\right)^{-a}\left(\frac{\psi}{\Lambda}\right)^{-b}. \tag{64}$$

Then the remaining equations read

$$\nabla_r^2\phi = -\frac{aq^2}{2r^4\Lambda}\left(\frac{\phi}{\Lambda}\right)^{-a-1}\left(\frac{\psi}{\Lambda}\right)^{-b} + m_\phi^2\phi \tag{65}$$

and

$$\nabla_r^2\psi = -\frac{bq^2}{2r^4\Lambda}\left(\frac{\phi}{\Lambda}\right)^{-a}\left(\frac{\psi}{\Lambda}\right)^{-b-1} + m_\psi^2\psi. \tag{66}$$

Unfortunately, we cannot solve this set of differential equations analytically. However, because of the fact that the model (4) is usually considered in connection with low energy QCD we are mainly interested in the long range behavior of the solutions. In fact, the asymptotic form of the solution for $r \to \infty$ is found to be

$$\phi(r) = G_\phi\left(\frac{1}{r}\right)^{\frac{4}{2+a+b}} \tag{67}$$

and

$$\psi(r) = G_\psi\left(\frac{1}{r}\right)^{\frac{4}{2+a+b}}, \tag{68}$$

where the constants read

$$G_\phi = \left(\Lambda^{a+b-1}\frac{aq^2}{2m_\phi^2}\left(\frac{b}{a}\frac{m_\phi^2}{m_\psi^2}\right)^{-\frac{b}{2}}\right)^{\frac{1}{2+a+b}} \tag{69}$$

and

$$G_\psi = \left(\Lambda^{a+b-1}\frac{bq^2}{2m_\psi^2}\left(\frac{a}{b}\frac{m_\psi^2}{m_\phi^2}\right)^{-\frac{a}{2}}\right)^{\frac{1}{2+a+b}}. \tag{70}$$

Using (64) we easily obtain the resulting electric field

$$E(r) = q\frac{G_\phi^{-a}G_\psi^{-b}}{\Lambda^{-a-b}}\left(\frac{1}{r^2}\right)^{\frac{2-a-b}{2+a+b}} \tag{71}$$

and the corresponding electric potential

$$U(r) = q\frac{2+a+b}{3(a+b)-2}\frac{G_\phi^{-a}G_\psi^{-b}}{\Lambda^{-a-b}}\left(\frac{1}{r}\right)^{\frac{2-3a-3b}{2+a+b}}. \tag{72}$$

One can repeat the previous calculations and conclude that this configuration of fields has an infinite energy. For $a+b > \frac{2}{3}$ the total energy diverges due to the behavior of the fields at spatial infinity. Thus, the model (4) simulates the confinement of external electric source, where the confining potential is given by (72). As it was mentioned before, this potential should not depend on r stronger than linearly. This provides an upper bound for our parameters: $a+b < 2$. Finally, the massive scalar model describes confinement for the following parameters:

$$a+b \in \left[\frac{2}{3}, 2\right]. \tag{73}$$

In the very special case, when $a+b=2$ and the masses of the scalars are identical, $m_\phi^2 = m_\psi^2 = m^2$, we are able to find the generalized Dick analytical solution

$$\phi = \frac{1}{r}\sqrt{\sqrt{\frac{a}{2}}\frac{q}{m} + \left(\beta_0^2 - \sqrt{\frac{a}{2}}\frac{q}{m}\right)\mathrm{e}^{-2mr}} \tag{74}$$

and

$$\psi = \frac{1}{r}\left(\frac{b}{a}\right)^{\frac{1}{2+b}}\sqrt{\sqrt{\frac{a}{2}}\frac{q}{m} + \left(\beta_0^2 - \sqrt{\frac{a}{2}}\frac{q}{m}\right)\mathrm{e}^{-2mr}}. \tag{75}$$

In agreement with (73) such a form of the scalar fields guarantees the linear dependence of the electric potential at large distance. In fact, the electric potential reads

$$U = \frac{1}{q}\sqrt{\frac{2}{a}}\left(\frac{a}{b}\right)^{\frac{b}{b+2}}\ln\left(\mathrm{e}^{2mr} - 1 + \beta_0^2\frac{m}{q}\sqrt{\frac{2}{a}}\right), \tag{76}$$

and for $r \to \infty$ it diverges linearly.

In [6] Dick and Fulchert have proposed a model where the single scalar field ϕ representing the lightest 0^{++} glueball is linearly coupled to the gauge fields, i.e. $\phi F^a_{\mu\nu}F^{a\mu\nu}$. This corresponds to $a=1$ and $b=0$ in our case. This particular form of the coupling has been chosen in analogy to the chiral quark model [15]. The confining potential derived from their model reads $r^{\frac{1}{3}}$. However, one can notice that there is another particle which is relevant in the low energy regime – a scalar meson. It seems to be reasonable to assume that both scalars inflect the low energy dynamics. Due to that one should consider these scalar fields together. Then the natural generalization of the Dick–Fulchert model is a theory with one glueball–meson coupling, that is $\phi\psi F^a_{\mu\nu}F^{a\mu\nu}$ ($a=1$, $b=1$ in our model). It is very interesting that this form of interaction ensures also the linear electric potential. This unexpected and striking result can suggest that the correct low energy effective action should consist of more than only one scalar field.

5 The non-dynamical scalar fields

In the previous sections we observed that the dynamical scalar fields (massive as well as massless) can strongly modify the electric and magnetic solutions. Now we will show that these fields can play a very non-trivial role even when we would drop the kinetic term for the scalar [16]. As will be clarified later a model with non-dynamical scalar fields can be also interesting in the context of the low energy gluodynamics.

Let us start with the mixed case: a dynamical field ϕ and a non-dynamical ψ. Additionally, we assume that the potential for the non-dynamical field is analogous to that in the formula (36). Then the Lagrangian (in the simplest version) reads

$$L = -\frac{1}{4}\left(\frac{\phi}{\Lambda}\right)^a \left(\frac{\psi}{\Lambda}\right)^b F^{c\mu\nu}F^c_{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \alpha\Lambda^4\left(\frac{\psi}{\Lambda}\right)^d, \tag{77}$$

where d is a positive parameter. We will analyze the Coulomb problem. Then the pertinent equations of motion take the following form:

$$\left[r^2\left(\frac{\phi}{\Lambda}\right)^a \left(\frac{\psi}{\Lambda}\right)^b E\right]' = 4\pi q\delta(r) \tag{78}$$

and

$$\nabla_r^2\phi = -\frac{aE^2}{2\Lambda}\left(\frac{\phi}{\Lambda}\right)^{a-1}\left(\frac{\psi}{\Lambda}\right)^b, \tag{79}$$

$$\alpha d\Lambda^3\left(\frac{\psi}{\Lambda}\right)^{d-1} = \frac{bE^2}{2\Lambda}\left(\frac{\phi}{\Lambda}\right)^a\left(\frac{\psi}{\Lambda}\right)^{b-1}. \tag{80}$$

Obviously, the variation with respect to the non-dynamical field gives a constraint on the scalar field ψ; see (80). We can solve it and express ψ in terms of the dynamical fields ϕ and E:

$$\left(\frac{\psi}{\Lambda}\right) = \left(\frac{bE^2}{2\alpha d\Lambda^4}\left(\frac{\phi}{\Lambda}\right)^a\right)^{\frac{1}{d-b}}. \tag{81}$$

Then the remaining field equations are

$$\left[r^2\left(\frac{\phi}{\Lambda}\right)^{\frac{ad}{d-b}}\left(\frac{bE^2}{2\alpha d\Lambda^4}\right)^{\frac{b}{d-b}} E\right]' = 4\pi q\delta(r) \tag{82}$$

and

$$\nabla_r^2\phi = -\frac{aE^2}{2\Lambda}\left(\frac{bE^2}{2\alpha d\Lambda^4}\right)^{\frac{b}{d-b}}\left(\frac{\phi}{\Lambda}\right)^{\frac{ad}{d-b}-1}. \tag{83}$$

Finally, we obtain the following solutions:

$$\phi(r) = H\Lambda\left(\frac{1}{r\Lambda}\right)^{\frac{4d}{ad+2(d+b)}} \tag{84}$$

and

$$E(r) = \Lambda^2 q^{\frac{d-b}{d+b}}\left(\frac{b}{2\alpha d}\right)^{-\frac{b}{d+b}} H^{-\frac{ad}{d+b}}\left(\frac{1}{r\Lambda}\right)^{\frac{4(d-b)}{2(d+b)+ad}}. \tag{85}$$

Here the constant H takes the form

$$H = q^{\frac{2d}{ad+2(d+b)}}\left(\frac{b}{2\alpha d}\right)^{-\frac{b}{ad+2(d+b)}} \times \left(\frac{a}{4}\frac{(ad+2(d+b))^2}{(d-b)(ad+4b)}\right)^{\frac{b+d}{ad+2(d+b)}}. \tag{86}$$

The corresponding electric potential reads

$$U(r) = \Lambda q^{\frac{d-b}{d+b}}\frac{ad+2(d+b)}{ad-2(d+b)}\left(\frac{b}{2\alpha d}\right)^{-\frac{b}{d+b}} H^{-\frac{ad}{d+b}} \times \left(\frac{1}{r\Lambda}\right)^{\frac{ad-2(d+b)}{ad+2(d+b)}}. \tag{87}$$

It has a confining-like behavior if the parameters fulfill the following condition:

$$1 > \frac{2(d+b)-ad}{2(d+b)+ad} > 0. \tag{88}$$

On the other hand, because of the fact that the non-dynamical field plays the role of the Lagrange multiplier, the action can be rewritten in terms of the dynamical fields only. The new Lagrangian has the form

$$L = -\frac{1}{4}\left(\frac{\phi}{\Lambda}\right)^{\frac{ad}{d-b}}\left(\frac{F}{\Lambda}\right)^{\frac{b}{d-b}} F^{c\mu\nu}F^c_{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi, \tag{89}$$

where $F = \frac{1}{2}F^{c\mu\nu}F^c_{\mu\nu}$. One can easily check that, in the case of Coulomb problem, the pertinent equations of motion are identical to (82) and (83). The system with the dielectric function governed by a dynamical and a non-dynamical scalar field is equivalent to the model where the permittivity depends on the dynamical scalar as well as gauge fields. Thus, we have obtained a generalized version of the effective theory proposed by Pagels and Tomboulis a long time ago [17]. Contrary to the original version, (89) contains a non-minimal gauge–scalar coupling along with an exponent of the standard Yang–Mills invariant. This equivalence seems to be particularly interesting since the Pagels–Tomboulis model (and the similar Adler–Savvidy model [18–20]) reproduces the trace anomaly already on the classical level and gives a correct prediction for the confining potential.

Obviously, it is possible to obtain the standard Pagels–Tomboulis model treating also the second scalar field as non-dynamical. Let us assume that the potential for this field is

$$V(\phi) = \alpha'\left(\frac{\phi}{\Lambda}\right)^e,$$

where e is a new parameter and α' is a dimensionless constant. Then both scalars can be written as functions of the electric field. Finally, we obtain

$$L_{\mathrm{PT}} = -\frac{1}{4}\left(\frac{F}{\Lambda^4}\right)^{2\delta} F^c_{\mu\nu}F^{c\mu\nu}, \tag{90}$$

where

$$2\delta = \frac{ad+be}{ed-ad-be}.$$

As it was shown in [17, 21], this model guarantees the confinement of external, Abelian electric sources for

$$\delta \geq \frac{1}{4}. \tag{91}$$

The duality demonstrated above allows us to believe that an ultimate gluodynamics effective theory would share properties as correctly described by all instances presented in this paper.

6 Summary

The main aim of this work has been analyzing of the electric and magnetic solutions in the standard classical Yang–Mills theory coupled to two scalar fields. For particular values of the parameters of the discussed models (with massless, massive as well as non-dynamical scalar fields) we have obtained confining-like field configurations. External, static, electric sources have an infinite energy due to the long range behavior of the fields. From this point of view all models are very similar. Clearly, the values of the parameters are model dependent and different potentials known from fits to the phenomenological data are derived for different a and b. One can also notice that the model with massless scalars is not able to describe the linear electric potential which corresponds to the limit $a, b \to \infty$. In other words this model can only approximate the linear potential (but with arbitrary accuracy). Moreover, in contradiction to other Lagrangians, there are finite energy electric solutions in the massless scalar model. Here, such solutions representing screening phenomena on the classical level are rather non-physical. They are removed from the spectrum of the theory by adding a potential term for the massless fields.

In spite of the fact that the presented models in a similar way describe the confining solutions, they differ profoundly.

The main difference between our models has its source in the physical interpretation of the scalars. This is strongly connected with the glueball problem. Let us firstly discuss the simplest case, i.e. the model with the massive scalar fields. Then these fields can be identified with some massive particles relevant to the low energy sector of QCD. Evident candidates are the scalar glueball 0^{++} and a scalar meson. Both objects exist due to non-perturbative effects, confinement of the gauge and quark fields respectively. It follows that one could expect that these fields should appear in the "democratic" manner. They should couple with the gauge field identically: $a = b$. This gives us a prescription of how to correctly generalize the QCD motivated Dick–Fulcher model. Namely, instead of one glueball coupling we introduce one glueball–meson coupling. Surprisingly, such a form of the dielectric function provides the standard linear electric potential. For identical masses of the scalar glueball and meson the analytical solution has been explicitly found.

In the case of the massless fields the situation is a little bit more subtle. Massless scalars should rather not be identified with any particles. There are not massless particles in the low energy sector of QCD. Even though scalars do not possess a particle-like interpretation the existence of the glueballs is not excluded. There are at least two possible ways of introducing glueballs in the framework of the massless scalar model. Firstly, they can appear as finite energy magnetic solutions. However, such configurations exist simultaneously with electric screening solutions. If we get rid of the screening solutions by adding a potential term to the Lagrangian then also the finite energy magnetic solutions disappear. Unfortunately, we are not able to preserve this glueball-like sector and remove the screening sector. On the other hand one could try to find the glueball spectrum in a way analogous to what has been done for the charmonium and bottomium states, i.e. by means of the Schrödinger equation. In this picture the glueball would consist of a magnetic monopole and anti-monopole. Then the potential in the Schrödinger equation would be a potential between magnetic monopoles [22]. A similar glueball model has recently been considered in [23].

Even more interesting from the glueball point of view is the model with the non-dynamical scalar field (or fields). Such a model corresponds to the generalized Pagels–Tomboulis theory. Here glueballs could appear as toroidal solitons with a non-trivial Hopf index [24]. It follows from the observation that the restricted version of the Pagels–Tomboulis model with $\delta = -\frac{1}{4}$ possesses such solutions [25] (but then electric sources are not confined). We expect that an additional scalar field could join the glueball and confining sectors.

To conclude, all models presented here can be treated as candidates for the effective model for low energy QCD. They describe confinement and give reasonable quark–anti-quark potentials. However, the problem of the glueballs has not been satisfactorily solved yet. We would like to analyze this in our forthcoming paper.

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